Activity: Thin-Plate Bridge

Introduction

The structural stiffness of a member (represented by Young's Modulus, E, and the Moment of Inertia, I) are directly related to the deflections a member will undergo when loaded and are closely related to its strength. The point of this exercise is to demonstrate how the cross-sectional shape of a member controls its strength and stiffness.

Basic Activity: Creating a Bridge from a Sheet

There are different materials that can be used for this activity: paper, wood, or metal.

Materials Needed

- Option 1
  - Paper index cards (1 pack of 4" x 6" notecards), sheets of paper (8.5"x11"), or cardboard
  - Clear adhesive tape (1 roll)
  - Some kind of supports with same height (e.g. two thick books)
  - Some kind of weight to load (e.g. pennies)
- Option 2
  - Balsa wood sheet (1 sheet of 3" wide or 6" wide balsa wood)
  - Hot glue
  - Some kind of supports with same height (e.g. two thick books)
  - Some kind of weight to load (e.g. pennies)
- Option 3
  - Sheet metal (1 sheet thin enough to bend)
  - Some kind of supports with same height (e.g. two thick books)
  - Some kind of weight to load (e.g. pennies)

Procedure

1. Demonstrate Weakness of Sheet
   a. Place supports appropriate distance apart
   b. Place single sheet across supports
   c. Load with weight → shows weakness of sheets loaded along weak axis
2. Student Exploration of Solution
   a. Break the students into teams of two or three (you can have them come up with a team name)
   b. Give students each multiple sheets with the appropriate adhesive materials
   c. Tell them to try and create member out of materials that will hold weight
   d. Give them time to explore ways in small groups
   e. Let them explain how they came up with idea
   f. Test the sections (take videos during the failure of the section to allow students to see slow motion failures afterward)
3. Showing Real-Life Solutions

   a. Take the same materials you gave students
   b. Create an I-section or a box-section out of the materials and show its strength (you will likely need to provide transverse stiffeners)
   c. Show pictures of I-sections that are found in the real world

Possibilities Modifications

1. *Weight Competition*: give all the student groups the same materials and see who can create something that holds the most weight
2. *Deflection Measurement*: deflections can be measured and compared to calculated deflections. The deflections could be measured as weight is applied. A load versus deflection plot can be created on graph paper.

Possible Rubric for Scoring

If you make a competition out of the activity, the following point scale could be used:

- Total weight held (10 points for most weight and -1 for each place after)
- Weight of beam (10 points for lightest beam and -1 for each place after)
- Selection of beam shape and explanation (1 to 5 depending on explanation)
- Aesthetics (1 to 5 depending on whether beam is aesthetically pleasing)

Points to Emphasize

There are several things that can be highlighted in this activity:

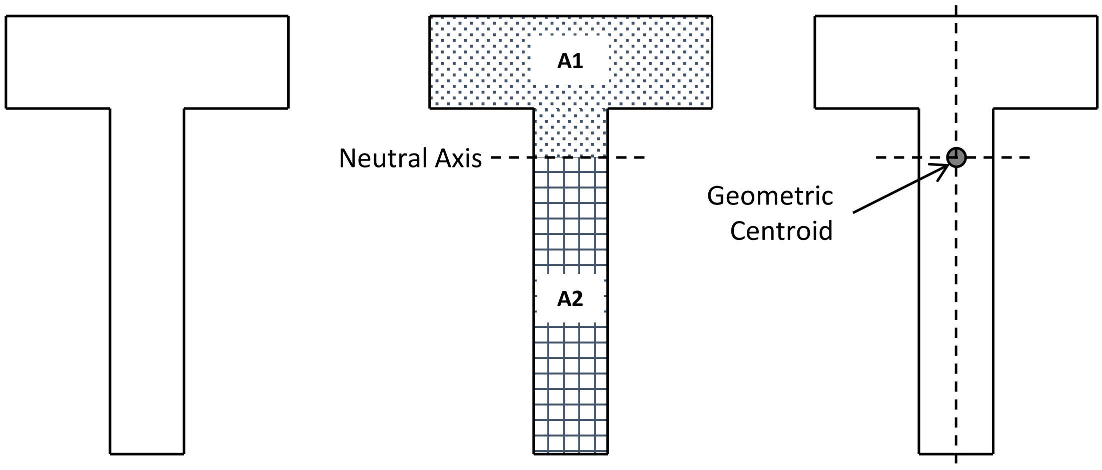
- *Effect of Cross-Section Shape on Stiffness*: The center of gravity is the horizontal line in the section which splits the section in half by area (i.e. there is an equal area below and above). A cross-section will be stiffer for sections with a larger area further away from the center of gravity. Students can draw their cross-section (with dimensions) and find the center of gravity using geometry and algebra.
- *Effect of Different Material Types on Stiffness*: A member's stiffness is dependent on both the section shape and the material. If you have access to several different material types, you can show how different materials will affect the stiffness.

Challenge Activities

1. Create the stiffest section out of the "plate" and adhesive provided
2. Guess / calculate which one will be the strongest before testing them
3. Give students opportunity to improve design after initial testing

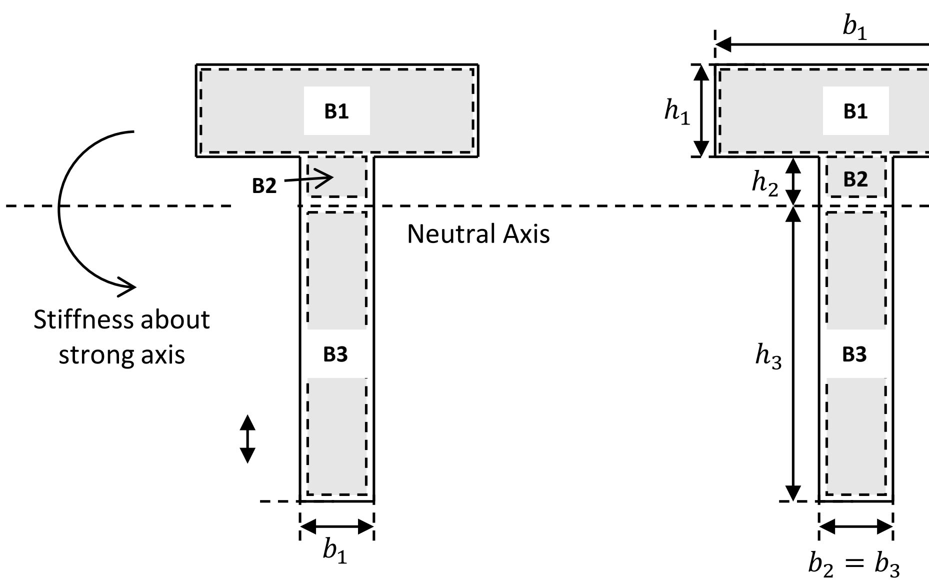
Math Applications

1. Finding the geometric centroid of a section



Find the position of the neutral axis such that A1 = A2

2. Comparing strength of sections through moment of inertia (finding the moment of inertia of a section)



Moment of Inertia
(for one rectangle)

$$I_g = \frac{1}{12} bh^3$$

For Multiple Rectangles
(Parallel-Axis Theorem)

$$I = \sum I_g + Ad^2$$

The section that we have been looking at is made up of three different rectangular sections (as shown above: B1, B2, and B3). We can then find the total moment of inertia of the section as:

$$I = \overbrace{\frac{1}{12}b_1{h_1}^3 + b_1h_1\left(\frac{h_1}{2}+h_2\right)^2}^{\text{Section B1}} + \overbrace{\frac{1}{12}b_2{h_2}^3 + b_2h_2\left(\frac{h_2}{2}\right)^2}^{\text{Section B2}} + \overbrace{\frac{1}{12}b_3{h_3}^3 + b_3h_3\left(\frac{h_3}{2}\right)^2}^{\text{Section B3}}$$

Students can use the above procedure to find the approximate moment of inertia of sections. This can be used to guess whose beam will be the strongest or help the students decide what section they want to make.

The moment of inertia and parallel axis theorem are derived expressions. More information can be found online if desired. There are different formulas for other section shapes that can be found online as well.

3. Finding deflections

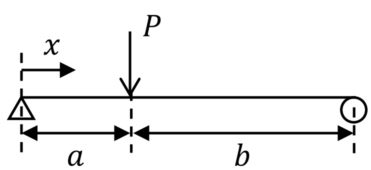
Deflections are related to the load applied by a form of the following relationship:

$$EI\frac{d^4\delta}{dx^4} = -q$$

where E = material stiffness, I = moment of inertia, δ = deflections, and q = applied load. This means that the deflections can be found by integrating several times:

$$\delta = \iiint \int \frac{q}{EI}$$

When a point load, P, is placed anywhere on the beam, this integral can be simplified to the below solution:



$$\Delta_x = \frac{Pax}{6LEI}(L^2 - a^2 - x^2)$$

This expression can be used to estimate the deflections that your beam will experience under any point load (P).

Approximate material stiffness properties can be found online (for whatever material you are using for the activity); it will likely be call the "Modulus of Elasticity". Ensure that all your units are consistent.